

## PROBLEM SETS 1 & 2. DUE THURSDAY 7 SEPTEMBER

### PROBLEM SET 1. PROBLEMS FROM LECTURE 1.

**Reading.** *Quick Calculus*, Chapter 1.

**Supplementary reading.** Simmons, Chapter 1.

1. Given a quadratic equation of the form  $ax^2 + bx + c = 0$ , we can solve for  $x$  using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Using the above formula, solve the equation  $4x^2 - 5x - 6 = 0$  for  $x$ .

2. Find  $f(x)$  if  $f(x+1) = x^2 - 5x + 3$ .
3. Graph the following functions, and give their domain and range.
  - (a)  $y = 3x - 2$ .
  - (b)  $y = 4x^2 + 3$ .
4. Graph the following functions, and give their domain and range.
  - (a)  $y = 2^x$ .
  - (b)  $y = 2^{x+3}$ .
5. Graph the following functions, and give their domain and range.
  - (a)  $y = \log_2(x)$ .
  - (b)  $y = \log_2(x) + 5$ .
6. Graph the following functions, and give their domain and range.
  - (a)  $y = \sin(x)$ .
  - (b)  $y = \sin(2x)$ .
7. Graph the following functions, and give their domain and range.
  - (a)  $y = \tan(x)$ .
  - (b)  $y = 4 \tan(x)$ .
8. Simplify the following expressions.
  - (a)  $\log_{10}\left(\frac{x+y}{z}\right)$ .
  - (b)  $25^{\log_{25}(x+y) + \log_5(\frac{x}{y})}$ .
9. Make the following computations using right triangles.
  - (a) For  $\theta = \frac{\pi}{4} = 45^\circ$ , compute  $\sin(\theta)$ ,  $\cos(\theta)$ , and  $\tan(\theta)$ .
  - (b) For  $\theta = \frac{\pi}{6} = 30^\circ$ , compute  $\sec(\theta)$ ,  $\csc(\theta)$ , and  $\cot(\theta)$ .
10. Simplify the following expressions (i.e. write them in terms of elementary trig functions  $\sin(\phi)$ ,  $\sin(\theta)$ , etc.).
  - (a)  $\sin(\theta + \phi)$ .
  - (b)  $\cos(3\theta)$ .

## PROBLEM SET 2. PROBLEMS FROM LECTURE 2.

**Reading.** *Quick Calculus*, pp. 50–97.

**Supplementary reading.** Simmons, Chapter 2, sections 2.1–2.5. Read section 2.6 if you are interested in some applications of the derivative.

- Compute the following limits.
  - $\lim_{\theta \rightarrow 0} \frac{\sin(5\theta)}{\theta}$
  - $\lim_{\theta \rightarrow 0} \frac{\sin(3\theta)}{\sin(4\theta)}$
  - $\lim_{x \rightarrow \infty} \frac{x}{x^2+1}$
  - $\lim_{x \rightarrow \infty} \frac{2x^2+3x}{3x^2-2}$
- Where are the following functions discontinuous?
  - $\frac{x}{x^2+1}$
  - $\frac{1}{x^2+x-6}$
  - $\frac{x^3+x}{x^2+1}$
  - $\frac{x^2+2x}{x^3+2x^2-x-2}$
- Use the definition of the derivative to show that for  $f(x) = ax^2 + bx + c$ , for constants  $a, b, c \in \mathbb{R}$ , the derivative is  $f'(x) = 2ax + b$ .
- Use the definition of the derivative to find the derivative of the function  $f(x) = \frac{x}{x+1}$ .
- Use the definition of the derivative and the double angle formula to compute the derivative of  $f(\theta) = \cos(\theta)$ .
- Sketch the graph of the following two functions. For each, state where it is **not** differentiable.
  - $f(x) = \sqrt{|x|}$ .
  - $f(x) = |x^2 - 9|$ .
- Let  $f(x) = \begin{cases} x^2 & \text{if } x \leq -1, \\ mx + b & \text{if } x > -1. \end{cases}$  What must  $m$  and  $b$  be for  $f(x)$  to be differentiable at all points?
- A penny is dropped off a ledge on the World Trade Center in New York City. The ledge is 1024 feet above the ground. The penny falls a distance of  $s = 16t^2$  feet in  $t$  seconds.
  - How long does the penny fall before it hits the ground?
  - What is the average velocity at which the penny falls during the first three seconds?
- With the same situation as in Problem [?], answer the following questions.
  - What is the average velocity at which the penny falls during the last four seconds?
  - What is the instantaneous velocity of the penny when it hits the ground?
- An oil tank is to be drained for cleaning. There are  $V$  gallons of oil left in the tank after  $t$  minutes of draining, where  $V = 50(40 - t)^2$ .
  - What is the average rate at which oil drains out of the tank during the first 20 minutes?
  - What is the rate at which oil is flowing out of the tank 20 minutes after draining begins?